New Brunswick
Geometry and Applications in Mathematics 112

Statistics Unit Plan
&
Collection of Lesson Plans

Chris Adamson
January 23, 2011
Introduction

Teaching high school mathematics to me doesn’t simply entail a teacher writing questions and solutions on the board while students copy and listen to what the teacher lectures. My philosophy includes the former tasks; however, it is definitely not exclusive to them. I believe having conversations with students (by asking students questions) as appose to one-way lecturing enhances learning because it keeps students engaged. I also believe that allowing students to work in groups while learning mathematics is very advantageous to the strong students, the weak and the teacher. By having students active (coming to the board, getting into groups) versus sitting the entire time is more interesting for the students and allows them to be more connected. Lastly, I believe allowing students to attempt an example, summarize or describe what the teacher has just explained is essential in learning the concept and the research by Joan Middendorf & Alan Kalish (1996) support this. I demonstrate my philosophy of teaching high school mathematics within my two lesson plans, my unit plan and my unit test.

The kinds of tasks I give to students involves group work, discussions, coming to the board and marking their own quiz. By having students work in groups during class I help students develop an important life skill, promote shared ideas and peer to peer learning, which benefits both strong and weak students because teaching something is often a very powerful way of learning it better yourself. When I ask students to answer questions and discuss topics it encourages them to explain their reasoning and feel comfortable with not knowing the correct answer. Class discussion also allows students to hear the ideas and problems of others and this is important because chances are numerous students are struggling with the same problems. As a student approaches the board they feel a sense of confidence and power and this is something I would like my students to have. When the student is at the board a power shift from me to them occurs and this allows me to observe and allows students to become aware of how their peers tackle problems. A power shift also occurs when I allow students to immediately mark their own quizzes. This method provides immediate feedback when the topics are still fresh in the students’ minds.

I direct classroom conversation by first, actually having a conversation with my students as appose to lecturing. When I ask students a question, I no longer become the center of attention and the class is less teacher centered and therefore students become more engaged. I
will ensure correct content is discussed but will definitely allow students to express ideas, opinions and concerns. I intend my classroom to be an environment where students know their answer doesn’t have to be correct in order to be valued. I also plan to ask students to predict what they think before they come up with a solution. This increases their interest because they now want to know if their prediction was right. In both lesson plans I use the students as the data set. If the content is being learned by working with and discussing data based on the student’s responses, they are more likely to stay interested.

Assessment is accomplished by the use of formative assessments during class, homework, a quiz and a test. The formative assessment is built into my two lesson plans and is an assessment for learning. While students work in groups I will walk around and assess the language, knowledge and skills the students are using with a checklist. This will help me get a sense of how they’re doing and the assessments can be referred to later. When I assign a grade to homework assignments, students are more likely to complete them and therefore practice course material. As I look through student homework I can also get a sense of how each student is handling the material. Discussing the homework the following day encourages students to accomplish the homework before class in case I call upon them for an answer. This will also allow me to determine the work ethic of students. Immediate feedback occurs when students grade their own quizzes moments after they’ve completed them. Swift feedback is important for students because they can judge how well they’re doing while the topic is still fresh in their minds. The unit test students receive is fair and doesn’t include any content they haven’t been exposed to in class or homework.

Technology within this unit is mainly in the form of calculators so students can calculate faster and check their own math. A random number generator will be used on day 5 in order to demonstrate an understanding of the application of random numbers to statistical sampling. Technology could fit into future lesson plans, however doesn’t appear on the unit map.

Multiple representations are evident in my first lesson plan when I show students a chart containing the data they provided and later show them how to construct a histogram with the same data. This helps students understand, develop and communicate the same math but in different ways. In my second lesson plan and in the unit test I used different examples for a sample and a population and this shows students the same concepts can be applied anywhere. Lastly, in both lesson plans I physically move students into groups which represent samples
within a population. This is a great visual demonstration of the difference between sample and population.

I communicate my objectives with all stakeholders with all aspects of this collection. The lesson plans indicate my preparedness, my belief in conversation in the classroom, the curricular outcomes and how I will approach them. The formative assessments within the lessons indicate how I will be monitoring students throughout the unit and the unit plan shows how all curricular outcomes will be approached and achieved. Homework I assign to students tells them what kind of work I expect and by giving it a grade I can communicate with them by providing them feedback. The immediate grading of the quizzes also enables this. The combination everything previously mentioned in this paragraph will be beneficial when the time comes to discuss a particular student with any stakeholder.

Reference:
Course: Geometry and Applications in Math 112  
Name: Mr. C. Adamson  
Date: November 23  
Topic: F10- Interpret and Apply Histograms  
Unit: Statistics  
Grade: 11  
Materials: Pencil, paper, calculator

Intents/Objectives/Purpose:

Program of Studies (Pedagogic Purpose):
- Interpret data to determine the mean and median and construct histograms

Academic Purpose:
- Students will accurately construct histograms
- Students will correctly interpret data represented by histograms
- Students will determine the mean and median of data sets and differentiate between the two

Instructional Plan:

Introduction (5 minutes):
- Have the question “How many hours did you spend online last weekend?” written on the board when students arrive. Write examples underneath; Internet, Xbox Live, Online Gaming, etc.
- On the board beside the question have a table ready with the headings: Interval, Number of Students and Frequency.
  - Under the Interval column write 1-2, 3-4, 5-6, 7-8 and 9-10 in each row.
- Allow students to discuss amongst themselves while they decide upon their answer

Body (45 minutes):
- Begin marking down the responses from each student AND include yourself in the data
- While you explain the next bullet, have students copy this table and the rest of the example
- Explain the interval row is the possible number of hours spent online, the number of students row is for the number of students who spent that amount of time online and the frequency is another way of describing the number of students (10 minutes)
- Draw an x and y axis on the board
  - Ask students which axis frequency (y-axis) or interval (x-axis) should go on
  - Continue the discussion until students know the correct way to set up the histogram
- Label the axis properly and ensure students do the same on their own page before they begin drawing
- HINT to students that drawing a histogram is similar to drawing a bar graph and ask students to draw the histogram using the class data
  - Ask if a student would like to draw the histogram on the board while the others work on their own
  - Walk around the class and aid students during the construction of the histograms (formative assessment; make note on hw students are doing)
  - Be sure the example on the board is being done properly and guide that student because others are sure to follow the example (20 minutes)
- To show students the importance of histograms, Ask students what is easier to determine when looking at a histogram versus the chart

- Ask students again how many hours they spent online last weekend, but be sure to get the exact number, not the interval.
- Using the board WRITE out every student’s response
- Write the words mean and median on the board beside their responses
  - Ask students if they can explain each term
Continue the discussion until the class has defined both terms
- Mean is achieved by adding all numbers and dividing by the total number of students
- Median is the number in the middle; therefore numbers need to be arranged in order from least to greatest first

Have students determine the mean and median of the class data (calculators are allowed) (15 minutes)

**Closure (5 minutes):**
- Tell students the data they used in today’s class was a sample of the school’s student community
  - If the class data was a sample, ask students if they know what the whole school would be referred to as (a population)
- Ask them to think how the data provided by a sample can be different from the data provided by a population
  - This leads into the next lesson on “sample and population standard deviations”
- **BE SURE TO MARK DOWN THE CLASS MEAN**

**Assessment (5 minutes):**
- Provide students with the attached homework sheet
- Explain what bin-width is
- The homework requires students to interpret data, create histograms and find means and medians
- Students are required to hand their homework in the following day to the teacher
  - A strong understanding of mean is necessary for the next lesson

**Supporting Students:**

**Struggling Students:**
- As I walk around while students are constructing histograms I can ask them to explain why they are doing certain things and help them understand the steps in creating a histogram. By having an accurate example on the board, these students will also be able to check their work as they go.

**Enrichment:**
- By asking for a volunteer to draw their histogram on the board encourages stronger students to display their skills.
- When drawing the histogram and calculating the mean and median becomes effortless for some students, I can introduce the concept of standard deviation to them while other students are still working.
1. A local high school conducted a survey to determine how many students in each class use their agendas. The number of students using agendas per class are as follows: 4, 3, 7, 9, 5, 2, 7, 4, 8, 2, 9, 10, 12, 2, 14, 12, 3, 5, 9, 3, 16.
   a. Construct a histogram with a bin-width of 2.
   b. Determine the mean and the median for the data set.

2. The histogram below shows scores on the final exam for a group of students.
   a. Approximately how many students wrote this exam?
   b. Between what percentages did the majority of the students receive?
   c. What percent of students received between 0-20%? 40-60%?
<table>
<thead>
<tr>
<th>Course:</th>
<th>Geometry and Applications in Math 112</th>
<th>Name:</th>
<th>Mr. C. Adamson</th>
<th>Date:</th>
<th>November 24</th>
</tr>
</thead>
<tbody>
<tr>
<td>Topic:</td>
<td>F9- Demonstrate and understanding of the difference between sample standard deviation and population standard deviation</td>
<td>Unit:</td>
<td>Statistics</td>
<td>Grade:</td>
<td>11</td>
</tr>
<tr>
<td>Materials:</td>
<td>Pencil, paper, calculator</td>
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</tbody>
</table>

**Intents/Objectives/Purpose:**

**Program of Studies (Pedagogic Purpose):**
- Explore how a sample can be (and can’t be) representative of an entire population

**Academic Purpose:**
- Students will understand the difference between a sample and a population
- Students will understand the concept of standard deviation
- Students will use sample standard deviation and population standard deviation when appropriate
- Students will use proper notation ($\mu$, $\bar{X}$, $\sigma_x$, and $S_x$)

**Instructional Plan:**

**Introduction (5 minutes):**
- Write the following groups on the board and have students sit in the designated groups
  - Group 1: 6 boys, Group 2: 6 girls, Group 3: 3 boys, 3 girls, Group 4: 3 boys, 3 girls
  - Make groups as close as possible to the above four
- Remind students of how they described their classroom as a sample within the school yesterday

**Body (45 minutes):**
- Once in groups, explain to students that each group is a sample of the classroom population
  - Explain that a sample is a subset of the larger population
  - Ask students why a sample is important in statistics & for examples of samples and populations
    - Make sure by the end of the discussion that students are aware samples are used because entire populations are too large to work with and therefore samples save time and money (5 minutes)
- Ask students to make a prediction on how close they think their group’s mean will be to the class mean
- Within the groups have students determine the mean of their group (formative assessment; make note)
  - Have a student from each group come to the board and write their group’s mean
  - Once all students have written their group’s mean, WRITE the class’ mean from yesterday
  - Ask students for possible reasons why some group means are closer to the class mean than others
    - Ensure to tell students samples do not always represent a population properly (10 minutes)
- Now that the group means (sample means) and class mean (population mean) are on the board ask the students the following:
  - If the group means are means of a sample, what should they be called?
  - And if the class mean is the mean of the population, what should it be called?
    - Make sure they know the difference between a sample mean and a population mean
- Ask students if they know what standard deviation is
  - Hint at what the word deviation means and what the class was just talking about (means)
    - After the discussion, WRITE the definition on the board: how much variation or "dispersion" there is from the mean (10 minutes)
Tell students the two types of standard deviation: **sample** and **population standard deviation**

- **Ask** students: “If standard deviation is a calculation of how much variation there is from the mean, which mean would we use to calculate sample standard deviation? What about population standard deviation?”
- Tell students that sample standard deviations are found using sample means and population standard deviations are calculated using population means
- COPY this chart onto the board and re-explain each component while you write: (10 minutes)

<table>
<thead>
<tr>
<th>Your Group (example)</th>
<th>“n” amount of people</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample</td>
<td>X₁, X₂, X₃, X₄, X₅, X₆…Xₙ</td>
</tr>
<tr>
<td>Sample Mean (X̄)</td>
<td>X₁ + X₂ + X₃ + X₄ + X₅ + X₆… Xₙ/n</td>
</tr>
<tr>
<td>Sample Standard Deviation (Sₓ)</td>
<td>Sₓ = √((X₁ − X̄)² + (X₂ − X̄)² +… + (Xₙ − X̄)²) / (n-1)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Your Class (example)</th>
<th>“n” amount of people</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population</td>
<td>X₁, X₂, X₃, X₄, X₅, X₆…Xₙ</td>
</tr>
<tr>
<td>Population Mean (μ)</td>
<td>X₁ + X₂ + X₃ + X₄ + X₅ + X₆… Xₙ/n</td>
</tr>
<tr>
<td>Population Standard Deviation (σₓ)</td>
<td>σₓ = √((X₁ − μ)² + (X₂ − μ)² +… + (Xₙ − μ)²) / n</td>
</tr>
</tbody>
</table>

- Tell students the only thing that they haven’t gone over today on this chart are the formulas for the standard deviations
  - **POINT** out that the only difference between the two formulas are the **means used** (sample versus population) and the **denominator** (n-1 for sample and n for population)
- Let students know this chart sums up today’s class (10 minutes)

**Closure** (10 minutes):
- Using one of the group’s data, **DEMONSTRATE** to the class how to calculate the sample standard deviation
- **Ask** students to think about why calculating means and associated standard deviations are important and be prepared to discuss the topic in tomorrow’s lecture
  - This leads into the next lesson on “normal distributions”

**Assessment:**
- Provide students with the attached homework sheet
- The homework requires students to differentiate between a sample and population and to calculate the mean and standard deviation for each
- Homework will be discussed the following day prior to the question asked during the closure
  - A strong understanding of sample and population means and standard deviations are necessary for the next lesson on normal distributions

**Supporting Students:**

**Struggling Students:**
- As students work in groups I can check on students that may be struggling with calculating the mean and walk them through it
- I can also provide different examples of what a sample and a population are (a handful of skittles versus
the whole bag) to help demonstrate the concept

- I can also make myself available during lunch hour for any students who feel they need some clarification on any topic. During this time I would ask students what they think and why and move from there (try to understand where they’re coming from first)

**Enrichment:**
- During the closure of the lesson I could ask if any students would like to demonstrate the calculation of their groups sample standard deviation or even the population standard deviation
1. A university with 12,000 students is interested in knowing how many hours per week students spend studying outside of class. Two hundred fifty students were selected at random and asked, “How many hours per week do you spend studying?”
   a) What is the population of interest?
   b) What group constitutes the sample?

2. A biologist is researching bats and their ability to detect insects within close proximity of them. The following data represents the distances (cm) at which 6 different bats first detected a nearby insect:

   62 23 27 56 52 34

   a) Is this a sample or a population? Explain.
   b) Without using your calculator, determine the standard deviation.
   c) Check your answer using a calculator.

3. Mr. Sweeney has a small class of 15 math students. He is interested in knowing the mean and standard deviation of the class on their quiz. Their marks are given below.

   82 76 65 78 81 90 52 93 50 89 70 85 59 60 52

   a) Explain why this group represents a population and not a sample.
   b) Calculate the mean and standard deviation of this data (calculators allowed).
   c) Calculate the median of this population.
<table>
<thead>
<tr>
<th>Day</th>
<th>Topic (Outcome)</th>
<th>Description (Students will...)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Interpret and apply histograms</td>
<td>• See first lesson plan</td>
</tr>
<tr>
<td>2</td>
<td>Demonstrate an understanding of the difference between sample standard deviation and population standard deviation</td>
<td>• See second lesson plan</td>
</tr>
</tbody>
</table>
| 3   | Apply characteristics of normal distributions      | • Know what “bell-shaped’ is  
• Know a normal distribution is symmetric around the mean and median  
• The mean and median of a normal distribution are equal  
• Be able to locate 1, 2 and 3 standard deviations way from the mean |
| 4   | Distinguish between descriptive and inferential statistics | • Draw inferences about a population based on a sample  
• Differentiate between descriptive and inferential statistics and use them accordingly                                                                                                                                       |
| 5   | Design and conduct surveys and/or simulate data collection to explore variability | • Demonstrate an understanding of the application of random numbers to statistical sampling by using a random number generator  
• Distinguish between a simulation and drawing a real sample from an actual population                                                                                                                                 |
| 6   | Identify bias in data collection, interpretation, and presentation | • Realize an unbiased sample is usually randomly generated  
• Investigate simple random sample, stratified random sample, cluster sample and systemic sample  
• Also look at convenience sample and voluntary response sample |
| 7   | QUIZ                                                | • Assessment of the material covered to date  
• Second portion of the class will be to correct and review the quiz                                                                                                                                                             |
| 8   | Demonstrate an understanding of how sample size affects the variation in sample results | • Identify that larger sample sizes produce less variability than small sample sizes  
• Will accomplish this by comparing the standard deviation a large and small sample                                                                                                                                             |
| 9   | Determine, interpret, and apply confidence intervals | • Understand what confidence intervals are and how they can be used to make inferences about the population  
• Be able to identify and understand a point estimate, an interval estimator, a confidence level, and a confidence interval                                                                                                         |
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| 10 | **Demonstrate an understanding of how a confidence level affects a confidence interval** | • Will work with three different confidence levels: 90%, 95%, and 99%  
• Will compare 90%, 95%, and 99% confidence interval for the same data set to explore effect of the confidence level on the confidence interval  
• Will understand that a higher confidence level will result in a larger interval for a given sample size |
| 11 | **Demonstrate an understanding of the role of the Central Limit Theorem in the development of confidence intervals** | • Know that the construction of confidence intervals is based on CLT  
• Demonstrate knowledge that the CTL states conditions under which the mean of a sufficiently large number of independent random variables, each with finite mean and variance, will be approximately normally distributed |
| 12 | **Distinguish between the calculation of confidence intervals for a known population mean versus an unknown population mean** | • Use appropriate methods of calculating confidence intervals when populations means are known or unknown |
| 13 | Review | • Go over the review and take questions from students |
| 14 | **UNIT TEST** | • See unit test |
1. The number of goals scored by each player (15 in total) of the town’s soccer team was recorded this past season and are displayed below:
3, 6, 2, 6, 5, 3, 5, 9, 10, 2, 2, 8, 7, 7, 10
From these numbers, determine the following:
   a.) The mean
   b.) The median
   c.) The population standard deviation
   d.) A sample standard deviation using the first 5 players
   e.) Construct a histogram with a bin-width of 2
2. Normal distributions have important characteristics.
   a.) Name 2 characteristics of a normal distribution
   b.) Sketch a normal distribution of a population below
      i.) Be sure to indicate where the following are located: Mean, 1, 2 and 3 standard deviations from the mean.

3. A professor at a university decides to investigate how many university undergraduate students “pull all-nighters” (they do not sleep some nights). The professor decides to only ask the students in his 3 first year biology classes whether or not they “pull all-nighters”.
   a.) The professor’s sample is biased. Explain why and how the professor could fix his sample.
   b.) What type of sample did the professor originally take?
   c.) If the professor did take a perfectly unbiased sample and decided to make predictions about other university undergraduate students based on his data, what kind of statistics would the professor be using?
4. The age of university students in a certain province is normally distributed with an age of 24.1 years and a standard deviation of 2.3 years.
   a.) At what value would you expect your sampling distribution to be centered?
   b.) What would you predict the standard deviation of your sampling distribution to be if the sample size was 50 university students?
   c.) If you were creating an approximate sampling distribution of the sample mean based on a sample size of 100, how would it be similar to the sampling distribution you described in part (b)? How would it be different?

5. A botanist collects a sample of 50 petals and measures the length of each. It is found that $\bar{x} = 5.55$ cm and $s_x = 0.57$ cm. She then reports that she is 95% confident that the average petal length is between 5.39 cm and 5.71 cm.
   a.) Identify the point estimate, the confidence interval, and the confidence level.
   b.) Explain what information the confidence interval gives about the population of iris petal length.
   c.) How would the length of a 99% confidence interval be different from that of a 95% confidence interval?
   d.) If the botanist collected a sample of 200 petals, what would happen to the confidence interval and the confidence level?
6. A random sample of 35 red pine trees was selected from a large forest containing 100,000 trees. The mean diameter was determined to be $\bar{X} = 25.3$ cm with $S_x = 3.6$ cm.

a.) If we were to create a 95% confidence interval for the mean diameter of red pine trees in this forest, would we be allowed to approximate $\sigma$ with $S_x$? Explain.

b.) Create a 95% confidence interval and explain its meaning.

c.) Can you make any conclusions about the mean diameter for all red pine trees in the forest? Why or why not?

7. Indicate two (2) characteristics of the central limit theorem (CTL).